(A1)

ECE 412 Problem Set

(1) Joint prob. mass function of two RVs $\tilde{X}$ & $\tilde{Y}$ is given in the table below

$\tilde{X}$ \ $\tilde{Y}$	a	b	c
1	k	2k	k
2	k	2k	k
3	4k	k	4k

→ p(x,y) table

a) Find k, note: $\sum p(x,y) = 1$

b) Compute $H(\tilde{X})$

c) Compute $H(\tilde{Y})$

d) Compute $H(\tilde{X} | \tilde{Y} = a)$

e) Compute $H(\tilde{X} | \tilde{Y})$

f) Find $I(\tilde{X}; \tilde{Y})$

(2) Show that the conditional entropy $H(\tilde{X}, \tilde{Y} | \tilde{Z})$ can be written as

$$H(\tilde{X}, \tilde{Y} | \tilde{Z}) = H(\tilde{X} | \tilde{Z}) + H(\tilde{Y} | \tilde{X}, \tilde{Z})$$

$$H(\tilde{X}, \tilde{Y} | \tilde{Z}) = -\sum p(x,y,z) \log p(x,y|z)$$

(3) $\tilde{X}$ → R.V. $R_{\tilde{X}} = \{a, b, c\}$

$Prob(\tilde{X} = a) = 1/2$ $Prob(\tilde{X} = b) = 1/4$ $Prob(\tilde{X} = c) = 1/4$

$p(x) \rightarrow p(a) = 1/2$ $p(b) = 1/4$ $p(c) = 1/4$

(#2)

a) $H(\tilde{X}) = ?$

b) $\tilde{X}_1, \tilde{X}_2, \ldots \tilde{X}_n$ are independent i.i.d with common $p(x)$

The typical sequence $x_1, x_2 \ldots x_n$ satisfy

$$2^{-n(H(\tilde{X})+\epsilon)} \leq p(x_1, x_2, \ldots x_n) \leq 2^{-n(H(\tilde{X})-\epsilon)}$$

For $n=4$ Find all the typical sequences

i.e, $x_1 x_2 x_3 x_4$ → determine all typical sequences

c) Using (b) find the typical set $A_\epsilon^{(4)}$

d) Find the prob. of typical set in (c)

(4) $R_X = \{a, b, c, d\}$ $\tilde{X}$ → disc. R.V.

$p(a) = 1/16$ $p(b) = 1/16$ $H(\tilde{X}) = ?$

$p(c) = 1/8$ $p(d) = 3/4$

a) $n=10$ Find the number of all sequences (different)

$x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}$

b) Find the number of typical sequences

c) Find the number of non-typical sequences

d) How many bits per-symbol you use to represent
- non-typical sequences
- typical sequences

A3

(5) Explain the following definitions

- Non-singular code
- Singular code
- Instantaneous code
- Prefix code

(6) $R_X = \{a, b, c, d\}$ $X \rightarrow$ R.V.

$p(a) = 1/32$ $p(b) = 1/32$ $p(c) = \frac{7}{16}$ $p(d) = \frac{1}{2}$

$H(X) = ?$ Find an instantaneous (prefix) code for a, b, c, d and compute its average length. Verify Kraft inequality for the founded code

(7) What does optimal code mean, please explain.

(8) $X \rightarrow$ R.V. $R_X = \{a, b, c, d\}$

$p(a) = 1/8$ $p(b) = 1/16$ $p(c) = \frac{5}{16}$ $p(d) = \frac{8}{16}$

Find Shannon code lengths for a, b, c, d

(8) $R_X = \{a, b, c, d, e\}$ $p(a) = 0.25$ $p(b) = 0.25$ $p(c) = 0.2$
$p(d) = 0.15$ $p(e) = 0.15$

a) Find a prefix code for a, b, c, d, e
compute its average length

(A4)

b) Find a Shannon code for a, b, c, d, e and compute its average length

c) Show that for both cases

$$L_{avg} > H(X)$$

probabilistic b average code length

d) Find Huffman code for a, b, c, d, e

(10) What does data compression mean?
Explain the philosophy behind data compression.